Technical Comments

Comments on "An Analytical Approach to Hypervelocity Impact"

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In a recent technical note, Yuan and Bloom[1] present what is intended to be a refinement of certain aspects of an earlier paper, their first reference,[2] and an argument for the validity of a result given in the earlier work. The present comments therefore apply in part also to the earlier paper.

The authors' technical note utilizes a typical analytic form for the shock pressure-density (Hugoniot) relation for a metallic solid in place of the perfect-gas equation of state used in the earlier work. The analytical approach employed both in the note and in the authors' first reference uses, as a starting point, the differential equations of one-dimensional, steady, compressible, viscous flow that, in fact, do not apply as they stand to the unsteady flow problem of impact. The correspondence with the impact problem seems to have been drawn on the basis of misinterpretation of the meaning of variables in the result of integration of the steady flow equations. Integration of the steady flow equations can, however, be used to study a plane-shock flow in a Newtonian viscous fluid continuum in a frame of reference in which the shock layer is stationary. In what follows, specific comments are given regarding the authors' treatment of the impact problem.

The symbol u is used first in the section on "Hypervelocity Impact Model" to denote the material velocity in the region between the opposite-facing shocks in the projectile and target in a frame of reference in which the undisturbed target material is at rest. For identical projectile and target materials in one-dimensional impact, no assumptions regarding flow steadiness, shock thickness, or fluid behavior of the materials are required to conclude that the contact-interface velocity in such a frame is exactly half the impact velocity V. This follows immediately from symmetry of the impact in a frame of reference moving at half the impact velocity.

The authors' Eq. (2) is a typical analytic form for the Hugoniot curve for a dense solid relating pressure and density changes through plane shocks between states of uniaxial strain. It is significant that the form of their Eq. (2) is derived from the linear relation $w = C + Su$ between shock speed w and material velocity jump u across the shock, the constants C and S being obtained from straight-line fits of velocity measurements in one-dimensional, plane-shock experiments.[3]

The authors' Eqs. (3–5) are the steady, one-dimensional flow equations for a Newtonian viscous compressible fluid, and, as such, they are not applicable as they stand to the unsteady-flow problem of impact. The authors, however, incorrectly identify the velocity u in these steady flow equations directly with the constant value $V/2$, which exists behind the shock in the target in a frame of reference in which the undisturbed target material is at rest.

A problem to which the authors' Eqs. (3–5) may actually be applicable is that of flow through a shock layer in a viscous fluid (e.g., that in the target) in a frame of reference traveling with the shock, assuming that the shock moves with constant velocity relative to undisturbed material and that the state variable distributions with respect to the position coordinate x are invariant with time t in the moving frame, thus defining a steady flow. In such a reference frame, the flow velocity at infinity ahead of the shock has the constant magnitude $C + SV/2$ directed toward the shock, and that which is behind has the magnitude $C + (S - 1)V/2$ away from it, if the flow is considered to have been established originally by single-material impact at an initial relative velocity V in a material characterized by the authors' Eq. (2).

In the steady shock-flow problem, described previously, which the authors' Eq. (7) actually may govern, $P = 0$ at infinity ahead of the shock, and $P = \rho_0(C + SV/2)V/2$ at infinity behind the shock, the latter condition being obtained from the jump condition for momentum conservation across the shock transition. Conditions on the velocity at infinity cited in the preceding paragraph would have to be applied to determine the constants of integration of their Eq. (7) in order to solve the steady shock-flow problem.

In the authors' first reference,[2] the steady flow equations had also been applied to the unsteady impact problem directly, the velocity in those equations being identified incorrectly with the constant material velocity $V/2$ behind the shock in an initially stationary target. In their first reference, however, a perfect-gas equation of state had been used to characterize the material. As discussed in foregoing comments, the steady flow equations actually can be applied to the problem of flow through a stationary viscous shock layer. It is interesting to note that Taylor[4] attacked this problem many years ago in order to obtain an estimate of shock thickness in a viscous gas within the framework of continuum flow theory.

As the starting point for the steady shock-flow analysis, the authors' Eqs. (3–5) may be combined with the perfect-gas equation of state to obtain, without further approximation, the differential equation

$$m\left(\frac{dI}{dx} + u\,\frac{du}{dx}\right) - \frac{4}{3}\frac{d}{dx}\left[\mu\left(\frac{\frac{3}{4}}{Pr}\frac{dI}{dx} + u\,\frac{du}{dx}\right)\right] = 0 \quad (1)$$

where $m = \rho u$ ($= C_1$ in the authors' notation) is the constant mass-flow rate, I is the specific enthalpy, and μ is the viscosity. Here $Pr = \mu c_p/k$ is the Prandtl number, in which c_p is the specific heat at constant pressure (at most a function of temperature only) and k is the thermal conductivity.[5] One integration gives

$$m\left(I + \frac{u^2}{2}\right) - \frac{4}{3}\,\mu\left(\frac{\frac{3}{4}}{Pr}\frac{dI}{dx} + u\,\frac{du}{dx}\right) = mI_0 \quad (2)$$

where the integration constant has been written for convenience as mI_0 ($= -C_1C_3$ in the notation of the authors' first reference). With the simplifying choice of $Pr = \frac{3}{4}$, Eq. (2) takes the form

$$df/dx = (3m/4\mu)f \quad (3)$$

where

$$f = (1/I_0)[I + (u^2/2)] - 1$$

The solution of Eq. (3) is

$$f = A\,\exp[(3m/4)\int (dx/\mu)] \quad (4)$$

where A is an integration constant.

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Now, to study a steady shock flow, one must require the flow variables to have finite constant values at infinity to either side of the shock layer. For finite values of μ, this in turn requires $A = 0$ and gives the result $I + u^2/2 = I_0$, so that the total enthalpy is constant throughout at the value I_0. One now can integrate the momentum equation, replacing the pressure P by its equivalent $(\gamma - 1)\rho I/\gamma$ through the perfect-gas equation of state, where γ is the specific-heat ratio, and expressing I in terms of u^2 and the constant I_0 from the result obtained previously. This finally leads to a relation between u and x only, in which the integration constants must be determined so as to meet the conditions on u at infinity.

In the authors' first reference, the equivalent of Eq. (1) was integrated to obtain Eq. (24) of their first reference. Thus

$$\frac{x}{d} = \frac{k}{C_1 c_p d} \ln\left[1 + \frac{1}{C_3}\left(\frac{u^2}{2} + c_p T\right)\right] \qquad (5)$$

which is just a rearranged form of the logarithm of Eq. (4) with μ and c_p each constant and with the integration constant A taken to be unity. Both sides of the preceding equation have been divided by d, which was intended to represent a projectile diameter. In addition to the incorrect association of the velocity u in this equation with half the impact velocity V, the temperature T was replaced by its strong-shock asymptotic value in terms of V.

In the authors' first reference, the constant C_3 and the coefficient of the logarithm on the right in the preceding equation [Eq. (5)] were regarded as parameters to be chosen so as to fit hypervelocity penetration data. The identification of the position variable x with the penetration depth in pellet impact on an infinite target is completely without foundation. After introduction of the square of the stress-wave speed in an elastic bar E/ρ in the authors' first reference, Eq. (5) was finally put into the form shown as Eq. (10) of the note under discussion.

As was remarked earlier, the integration of Eq. (1) is an essential preliminary step to the analysis of a problem to which the authors' Eqs. (3–5) actually apply, namely, that of steady flow through a stationary viscous shock layer in a gas. In that steady flow problem, integration of Eq. (1) simply leads to the conclusion that the total enthalpy of the flow is constant throughout, a fact useful in subsequent analysis in which the final solution for the velocity distribution is required to attain prescribed fixed values at infinity.

In obtaining an integral of Eq. (7) of the note under discussion, the authors again introduce a projectile diameter d by dividing both sides of the resulting equation by that quantity. Thus k_1 in their Eq. (8) contains the factor $1/d$, as does the coefficient of the logarithm in Eq. (5). Again, the authors incorrectly associate the independent position variable x in the one-dimensional, steady flow equations with the penetration depth p in the unsteady impact problem.

Neither the procedure in the note under discussion nor that in their first reference is susceptible to rationalization. The final fit of impact data (the authors' Fig. 5) is indeed surprisingly good, when it is considered that the analysis both in the note under discussion and in their first reference uses, as a starting point, differential equations that do not apply directly even to one-dimensional impact.

In summary, the authors have used an inapplicable theory to study the impact problem, arriving at an analytic expression that was fitted to penetration data by the adjustment of constants in the expression. A steady flow problem, which is associated with one-dimensional impact, and to which their work can actually be considered to apply, has been described in the foregoing discussion. The result of integration of their Eq. (7) possibly may be useful for obtaining an estimate of the thickness of a stationary shock layer in steady flow of a dense viscous material described by a single pressure-density equation, their Eq. (2), provided that the resulting velocity distribution can be made to meet prescribed fixed values at infinity consistent with the values of pressure there through the jump condition for over-all momentum conservation across the transition.

References

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Reply by Authors to T. A. Zaker

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In the preceding comment on Ref. 1, Zaker questioned the validity of the differential equations of one-dimensional, steady, compressible viscous flow as applied to the hypervelocity impact problem and made specific comments regarding the authors' treatment of the impact problem without, however, offering any suggestions as to how the problem should be treated. First of all, cratering and penetration of targets by projectiles traveling at hypervelocity are very complex physical phenomenon. In the region of hypervelocity impact, the properties of materials involved are beyond the present state of knowledge, and the equation of state for these materials has yet to be established. For these reasons, considerable confusion still exists in the field of hypervelocity impact.

In Ref. 2 a theory of crater formation in solids by hypervelocity impact is studied from the standpoint of radially symmetric advancing shock fronts. The differential equations of one-dimensional, unsteady flow of a compressible inviscid fluid in spherical coordinates lead to a solution based on progressing waves which leads to a $\frac{2}{5}$ power law for penetration vs velocity. Because of the mathematical difficulties that are present, the momentum condition cannot be satisfied by the solution. The results of this elegant analysis are not in good agreement with experimental data. Yuan and Courter[3] have recently analyzed this problem by using the differential equations of an unsteady flow of a viscous compressible fluid in one-dimensional spherical coordinates. Because of the unknown viscous properties of materials at the impact condition, the penetration-impact velocity solution cannot be determined as yet. It is clear that a workable expression for the pentration-impact velocity relation based on unsteady flow theory cannot be realized for sometime to come.

If one considers a system of coordinates moving along with the wave front, the steady flow condition exists in which the respective shocks are stationary. The motion of the fluid in the shock layer is then governed by the equations for one-dimensional steady flow of a viscous compressible fluid [Eqs.

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